Replacement Sensor Model Tagged Record Extensions Specification for NITF 2.1

APPENDIX A

draft

7/23/04

John Dolloff
Charles Taylor

BAE SYSTEMS
Mission Solutions
San Diego, CA

RSM Grid Interpolation

An image's RSM ground-to-image grid consists of an image point coordinate $I_j = [r \quad c]_j^T$ associated with each ground point coordinate $X_j = [x \quad y \quad z]_j^T$ located within a grid spanning the RSM ground domain for that image (or image section if there are multiple sections). The actual X_j coordinate values are not included, but are defined by various parameters such as a grid origin, fixed step sizes along each of the x, y, z coordinate axes, and the number of points per coordinate axis. These parameters are included in the image's RSM support data, as described in the RSMGGA TRE.

The ground-to-image function $grid(X)$ outputs an image point $I = [r \quad c]^T$ associated with an arbitrary ground point $X = [x \quad y \quad z]^T$. The function interpolates the I_j in a set of X_j surrounding X. Piece-wise local interpolation, rather than a global spline interpolation, is recommended for speed. In particular, separable tri-quadratic interpolation is a recommended technique, which uses a 3×3×3 grid of X_j surrounding X. The corresponding interpolating polynomial is of the general form:

$$r = \sum_{k=0}^{2} \sum_{j=0}^{2} \sum_{i=0}^{2} c_{ijk} x^i y^j z^k , \tag{A-1}$$

i.e., all cross-terms associated with the multiplication of three quadratic polynomials, one a function of x, one of y, and one of z. Fixed grid spacing in each of the three ground space dimensions is also required, as specified in the RSMGGA TRE. Collectively, these properties allow for an efficient algorithm for the generation and evaluation of the interpolating polynomial, as described below.

Three z -planes are shown in Figure A-1, each with a 3×3 grid in x and y. A two-dimensional interpolation is first done at the points in the z -planes indicated by the open circles. Then a one-dimensional interpolation is done along the line indicated for the resultant image coordinate (r) at the desired ground point position (X) indicated by the filled circle.

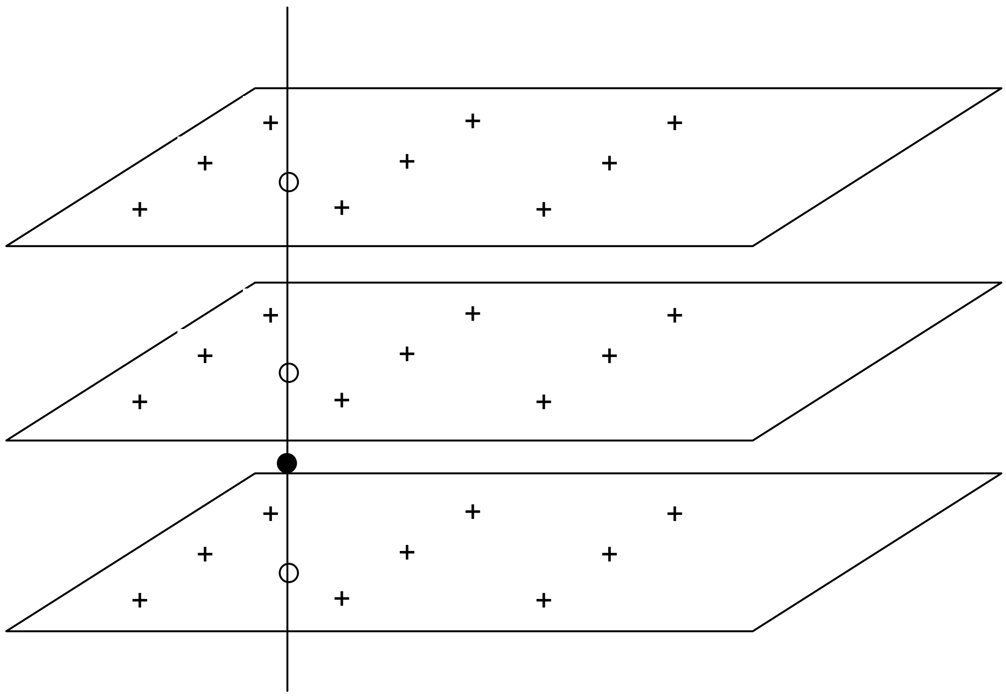

Figure A-1. Interpolation along the z -direction.

Separable interpolation can be done recursively, so that the two-dimensional interpolations in each plane are successively reduced to one-dimensional interpolations, as illustrated in Figure A-2. For a given plane, the procedure is as follows. First, a one-dimensional interpolation is done in the x -direction along each horizontal line, generating a value at the diamond (◊) using the values at each of the three cross (+) signs. Following these three one-dimensional interpolations, another is done in the y -direction along the vertical line containing the three diamonds. A value is generated at the desired open circle position using the values at each of the three diamonds.

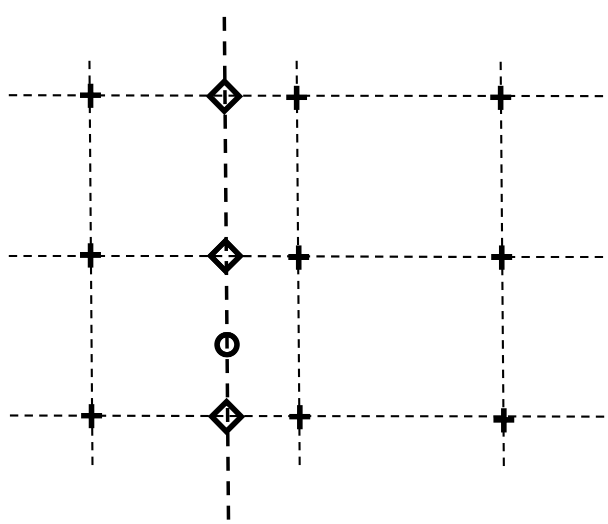

Figure A-2. Interpolation within a z-plane.

There are a total of four one-dimensional interpolations per plane, and a total of thirteen one-dimensional interpolations required for the final value u at the desired three-dimensional ground point position X. Each one-dimensional interpolation is second-order (quadratic), i.e., based on a quadratic polynomial in one variable. The general (Lagrange) formula for a one-dimensional, second-order interpolation of a value s at position t, using values s_i at three evenly spaced positions t_i (grid spacing d, and $t_{i-1} \le t \le t_{i+1}$), can be written as follows:

$$
\begin{aligned}
s = {} & s_{i-1}\left[-\frac{1}{2d}\left(t-t_i\right)+\frac{1}{2d^2}\left(t-t_i\right)^2\right] \\
& + s_i\left[1-\frac{1}{d^2}\left(t-t_i\right)^2\right] \\
& + s_{i+1}\left[\frac{1}{2d}\left(t-t_i\right)+\frac{1}{2d^2}\left(t-t_i\right)^2\right]
\end{aligned}
\tag{A-2}
$$

Note that s takes on the value s_{i-1}, s_i, and s_{i+1} at t_{i-1}, t_i, and t_{i+1}, respectively.

In our application, the independent variable t is either x, y, or z, and the dependent variable s is r. The values s_i are either r_i at the appropriate grid point position in the 3×3×3 grid, or a linear combination of r_i generated from previous one-dimensional interpolations. Note that all terms (weights) multiplying s_{i-1}, s_i, s_{i+1} are invariant across all nine one-dimensional interpolations in the x-direction, and all three one-dimensional interpolations in the y-direction performed in all three planes. Therefore, they need only be computed once per direction. Also, the overall algorithm will yield the same result r for a position X regardless the order of interpolation. For example, x-planes could be used instead of z-planes. And, of course, the entire procedure described above is also applicable to the image point c coordinate, i.e., there are actually two interpolating polynomials for a given X, one for r and one for c. Terms multiplying the data (s_{i-1}, s_i, s_{i+1}) are identical for their corresponding one-dimensional interpolations.

Analytic partial derivatives of the interpolating polynomial, and hence for the overall RSM ground-to-image function $grid(X)$, can also be generated in an efficient manner when required for geopositioning or triangulation solution algorithms. The above procedure that generates r at X using (thirteen) one-dimensional interpolations is modified in order to generate the partial derivative of r with respect to the desired ground coordinate component (x, y, or z) at the position X. The modification is minor: if a particular one-dimensional interpolation (of the thirteen) involves the desired ground coordinate component, simply replace Equation A-2 with the following equation, otherwise evaluate it as before.

$$\frac{\partial s}{\partial t} = s_{i-1}\left[-\frac{1}{2d} + \frac{1}{d^2}(t - t_i)\right] + s_i\left[-\frac{2}{d^2}(t - t_i)\right] + s_{i+1}\left[\frac{1}{2d} + \frac{1}{d^2}(t - t_i)\right] \tag{A-3}$$

The above description for interpolating polynomial generation and evaluation is not specific on how the actual $3\times3\times3$ grid is selected for a given X. It is recommended that the 3×3×3 grid be selected such that X is "centered". Specifically, such that for each dimension (x, y, z) the corresponding three data points are the nearest three data points.

In addition, there may be situations when a 3×3×3 grid may not be available for a particular X, such as that located at the boundary of the supplied RSM ground-to-image grid. In this case, linear interpolation/extrapolation is utilized, i.e., the one-dimensional interpolations are first order (linear).

Separable tri-cubic interpolation is also a recommended approach for the RSM ground-to-image function $grid(X)$. In fact, when the grid is not excessively dense, tri-cubic interpolation can be significantly more accurate than tri-quadratic interpolation in many situations.

The interpolating polynomial generation and evaluation procedure for separable tri-cubic interpolation is basically the same as that for separable tri-quadratic interpolation, except that a 4×4×4 grid is used, and the one-dimensional Lagrange interpolations are third order (cubic). For a given X, the $4\times4\times4$ grid is selected such that the point for interpolation is symmetrically located or "centered", i.e., in each dimension (x, y, z) the two closest data points are selected on either side.

The following presents the corresponding one-dimensional interpolation formula for a value s as a function of t, and its corresponding derivative with respect to t (grid spacing d, and $t_{i-1} \le t \le t_{i+2}$). They are directly analogous to equations (A-2) and (A-3) used for quadratic interpolation.

$$
\begin{aligned}
s = {} & s_{i-1}\left[-\frac{1}{3d}(t-t_i)+\frac{1}{2d^2}(t-t_i)^2-\frac{1}{6d^3}(t-t_i)^3\right] \\
& + s_i\left[1-\frac{1}{2d}(t-t_i)-\frac{1}{d^2}(t-t_i)^2+\frac{1}{2d^3}(t-t_i)^3\right] \\
& + s_{i+1}\left[\frac{1}{d}(t-t_i)+\frac{1}{2d^2}(t-t_i)^2-\frac{1}{2d^3}(t-t_i)^3\right] \\
& + s_{i+2}\left[-\frac{1}{6d}(t-t_i)+\frac{1}{6d^3}(t-t_i)^3\right]
\end{aligned} \tag{A-4}
$$

$$
\begin{aligned}
\frac{ds}{dt} = {} & s_{i-1}\left[-\frac{1}{3d}+\frac{1}{d^2}(t-t_i)-\frac{1}{2d^3}(t-t_i)^2\right] \\
& + s_i\left[-\frac{1}{2d}-\frac{2}{d^2}(t-t_i)+\frac{3}{2d^3}(t-t_i)^2\right] \\
& + s_{i+1}\left[\frac{1}{d}+\frac{1}{d^2}(t-t_i)-\frac{3}{2d^3}(t-t_i)^2\right] \\
& + s_{i+2}\left[-\frac{1}{6d}+\frac{1}{2d^3}(t-t_i)^2\right]
\end{aligned} \tag{A-5}
$$